

Midterm 2 Practice Problems

1. You are buying a Prius for \$25,000. In years 1-5, your gas costs will be \$600/year. Maintenance costs will be 0 in years 1-2 and then \$500 in both years 3 and 4 and then \$700 in year 5. Then, at the end of year 5, you will sell the car for \$13,000. What is the equivalent uniform annual cost of owning the Prius for this 5-year period? Your discount rate is 5%.

2. In which of the following is the assumption of repetition appropriate?

- a) Deciding whether to reopen a copper mine.
- b) Deciding whether to speed up R&D for a new drug.
- c) Deciding whether to replace the transmission on your car or to upgrade to a new car?
- d) Deciding whether to buy Dell or Apple laptops for your company.

3. You go for an oil change. Your mechanic offers to use premium oil (then the oil change would cost \$35 instead of \$30 for regular oil), which would mean you'd only need to come back in 9 mo. instead of 6 mo.

- a) Suppose the monthly discount rate is 0.5%. Determine whether it is more advantageous to use premium oil or regular oil. Assume an infinite horizon and repetition.
- b) Would it be more convenient if the discount rate in part b were a real or nominal discount rate? Explain.

4. A taxi company is considering buying taxis with diesel engines rather than gasoline engines. The cars average 50,000 mi / yr. Use a discount rate of 6%. Should you buy diesel or gas cars?

	Diesel	Gasoline
Vehicle cost	26,000	24,000
Useful life in years	3	4
Fuel cost per gal	3.84	4.08
Mileage (miles per gal)	42	29
Annual repairs	300	200
Annual insurance premium	500	500
End of life resale value	4,000	6,000

5. A mining company is considering two alternative ways of mining an ore deposit that it owns. The real discount rate is 9% and the table below contains projections of real cashflows, in millions of today's dollars. Alternative A is the strip mining approach examined in class; Alternative B would extract the ore by a slower, costlier, and less destructive method.

	Alternative A	Alternative B
Investment (Year 0)	30	100
Project Duration N	N = 10 years	N = 15 years

Annual Benefits (Years 1 through N)	60	35
Annual Costs (Years 1 through N)	10	12
Environmental Remediation Cost (Year N)	500	50

a) Calculate NPV and equivalent uniform annual worth (EUAW) for each alternative.

b) On economic grounds only, which one of the following would you recommend that the company do? In one sentence, explain why.

Alternative A

Alternative B

both Alternative A and Alternative B

neither Alternative A nor Alternative B

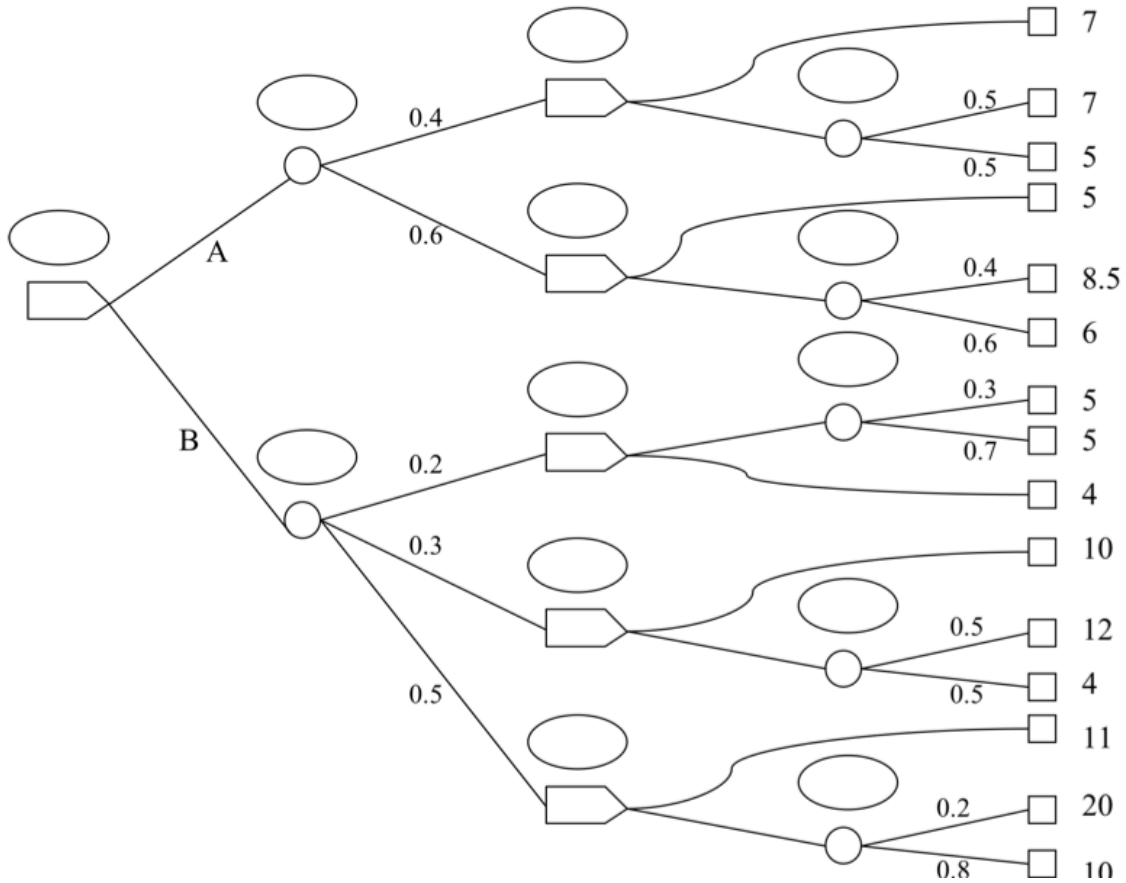
c) Suppose that competition decreased the price of ore (and thus the annual benefits) by X%. For alternative A, calculate the X at which point the project becomes uneconomic. Do the same for alternative B.

6. Consider the following decision tree. Solve the decision tree. Make sure to prune the sub-optimal branches.

a) Write the present value in the oval at each branching point.

b) What is the optimal initial decision (A or B)?

[Source: Kent Webb, Business 260]



7. Buzzy-B Toys must decide the course of action to follow in promoting a new whistling yo-yo. Initially, management must decide whether to market the yo-yo or to conduct a test marketing program. After test marketing the yo-yo, management must decide whether to abandon it or nationally distribute it.

A national success will increase profits by \$500,000, and a failure will reduce profits by \$100,000. Abandoning the product will not affect profits. The test marketing will cost Buzzy-B a further \$10,000.

If no test marketing is conducted, the probability for a national success is judged to be 0.45. The assumed probability for a favorable test marketing result is 0.50. The conditional probability for national success given favorable test marketing, is 0.80, for national success given unfavorable test results, it is 0.10.

a) Construct the decision tree. Make sure to label the nodes. Don't forget the probabilities.

b) Solve the decision tree. Make sure to prune the sub-optimal branches and label each node with its payoff (the net change in profit). (Hint: there is no discounting in this problem.)

c) Write a sentence explaining the optimal strategy.

[Source: Kent Webb, Business 260]

8. <http://people.brunel.ac.uk/~mastjib/jeb/or/decmore.html>

9. Suppose you plan to buy a car in 2 years. The model you want costs \$20,000 now, and you expect its price to rise with inflation, which will run at a constant annual rate of 3% over the next 2 years. You can invest in a 2-year CD (certificate of deposit) which pays 4% interest per year, compounded quarterly. How much do you need to invest now?

10. Ten (10) years ago Northwestern tuition was approximately 22,000 (excluding fees, room and board, etc.). Today (when I gave this problem) the tuition is 38,088. Assume the annual rate of inflation was 2% during this time period. At what annual **real** rate did tuition increase during this time?

11. Suppose you got a job in LA and want to buy a new car. This being LA you narrow your choice to either a Prius (costs \$25k) or a Miata convertible (costs \$20k). The Prius gets roughly 50 mpg (miles per gallon) while the Miata gets 25 mpg. Suppose you drive 15,000 miles a year.

a) Suppose gas is \$2/gal. Use a 5% discount rate. Which is the more economical choice assuming either car lasts 10 years? (We're ignoring salvage value, license fees, and repair costs here.)

b) Suppose gas is \$2/gal. today *and will increase 3% a year*. Use a 5% discount rate. Which is the more economical choice assuming either car lasts 10 years?

c) Suppose the Miata lasts 15 years while the Prius only lasts 10 years (because of the batteries). Assume gas is \$2/gal. today; the rate of inflation is 2%; and the real discount rate is 4%. What is the economical choice now? (Hint: this being LA, it's likely you'll still need a car in 10 or 15 years.)

12. You are working at the Chicago Housing Authority and need to choose among three alternatives for roofing on public housing projects. The following table gives costs, as of now, per 100 square feet of roofing, as well as lifespans. The Chicago Housing Authority uses a nominal discount rate of 6%. The inflation rate is 3%. Which of the three alternatives is best?

	Cheap Shingles	Expensive Shingles	Metal
purchase price	\$35	\$70	\$120
cost to install	\$70	\$70	\$110
annual maintenance	\$5	\$3	\$1
lifespan	15 years	25 years	50 years

13. To evaluate the NPV of the mortgage, is a real or actual discount rate more convenient? Explain.

14. Will the nominal payments on a mortgage increase over time with inflation?

15. Will the nominal rent payments increase over time with inflation (consider a long horizon, say 10 years)?

16. When would the actual discount rate be less than the real discount rate, $d_A < d_R$?

17. You're comparing tires with different lifespans.

Tire lifespan (years)	Cost
1	34.95
1.5	39.95
3	65.95
4	69.95
5	88.88

Your real discount rate is 2%, inflation is 1.7%.

a) What is the nominal discount rate?

b) Which tire should you choose?

18. Your savings account has an annual interest rate of 1.4% Inflation is 2% per year. What is your real interest rate?

19. The price of computers has decreased by about 1% a year in the last seven years. If inflation has been 2% a year, calculate the annual rate at which computer prices have changed in inflation-adjusted terms.

20. The city of San Francisco spends \$1m every year repainting the golden gate bridge. You have developed a paint that lasts twice as long. If the city were to switch to your new type of paint, how much would the city be willing to spend for each repainting? Assume the city uses a real discount rate of $d_R=4\%$ and a rate of inflation $f=2\%$.

21. You are buying a Prius for \$25,000 with a 5 year loan of 5 annual payments (the first payment being a year from today) with an interest rate i . Then, at the end of year 5, you will sell the car. A five year-old Prius is currently selling for \$13,000. Your gas consumption is about 100 gallons a year. The current cost of gas is \$4/gallon. You expect the car to need maintenance costs once, 3 years from today. The current cost for this maintenance is around \$500. You expect the inflation rate to be f . Draw the real and actual cashflow diagram leaving the amounts as formulas (either algebraic or Excel).

22. Your company manufactures iPhone cases. There are rumors that Apple will introduce the iPhone 5 in March. If you wait until the phone is announced to develop cases that fit it, you expect to make a profit of \$1m. If you make cases based on the rumors then there is a 60% chance that you make \$1.5m in profit because you will be able to sell your cases before your competitors. However, there is a 40% chance that

your cases won't fit and you will need to redevelop your cases after the phone is announced, giving you an expected profit of only \$300k.

a) Construct the decision tree. Make sure to label the nodes. Don't forget the probabilities.

b) Solve the decision tree. Make sure to prune the sub-optimal branches and label each node with its payoff. (*Hint: there is no discounting in this problem.*)

c) Write a sentence mentioning the expected profit and explaining the optimal strategy.

d) You are considering paying a reward of \$100k for more accurate information about the phone's dimensions. How accurate must the information be (i.e., with what probability will the cases fit), for this to be worthwhile? Construct the tree, solve it, and write a sentence explaining the solution.

Solutions

1. No inflation here. $NPV = -25k - PV(5\%, 5, -600) - 500 * 1.05^{-3} - 500 * 1.05^{-4} - 700 * 1.05^{-5} + 13k * 1.05^{-5} = -18,803.58$. So $EUAC = PMT(5\%, 5, NPV) = \$4,343.15$ per year.

2. d. For c, you won't repeatedly replace the transmission (other things will also fail).

3a. $r = 0.5\%$. $EUAC$ for regular oil = $30r / (1 - (1+r)^{-6}) = \5.09 .

$EUAC$ for premium oil = $35r / (1 - (1+r)^{-9}) = \3.99 .

Using premium oil is more advantageous.

3b. Real, because the cash-flows are inflation-adjusted (the nominal costs will increase over time).

4. $EUAC$ of Diesel is \$13,841.84. $EUAC$ of Gasoline is \$13,289.13. Gas is preferred.

5a. Alternative A: $NPV = -\$30 + \$50/(1+r) + \dots + \$50/(1+r)^{10} - \$500/(1+r)^{10}$

$$= -\$30 + \$50 * 1/r (1 - 1/(1+r)^{10}) - \$500 / (1+r)^{10} = \$79.68m$$

$$EUAW = NPV * r / (1 - 1/(1+r)^{10}) = \$12.42m$$

Alternative B: $NPV = -\$100 + \$23/(1+r) + \dots + \$23/(1+r)^{15} - \$50/(1+r)^{15}$

$$= -\$100 + \$23 * 1/r (1 - 1/(1+r)^{15}) - \$50 / (1+r)^{15} = \$71.67m$$

$$EUAW = NPV * r / (1 - 1/(1+r)^{15}) = \$8.89m$$

5b. These two projects are mutually exclusive so you can't do both. They can't be repeated so the right decision criterion is NPV. A has higher NPV and should be followed.

5c. Alternative A:

$$NPV = -\$30 + Y * 1/r (1 - 1/(1+r)^{10}) - \$500 / (1+r)^{10} = \$0m$$

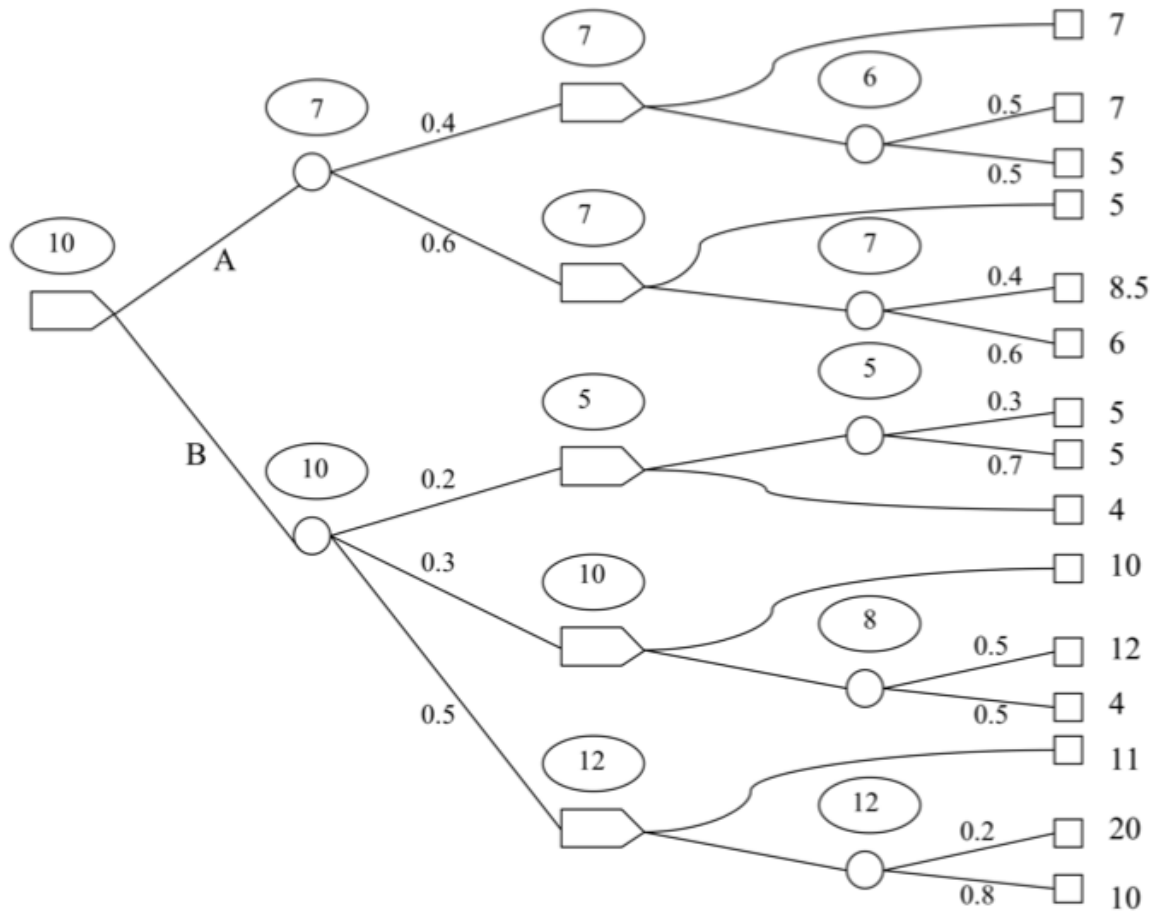
so, $Y = (30 + 500 / (1+r)^{10}) * r / (1 - 1/(1+r)^{10})$, where $Y = 60 * (1 - X\%) - 10$. So $X = 1 - (Y + 10) / 60$.

So $Y = 37.58$ and thus $X = 20.69\%$

Alternative B:

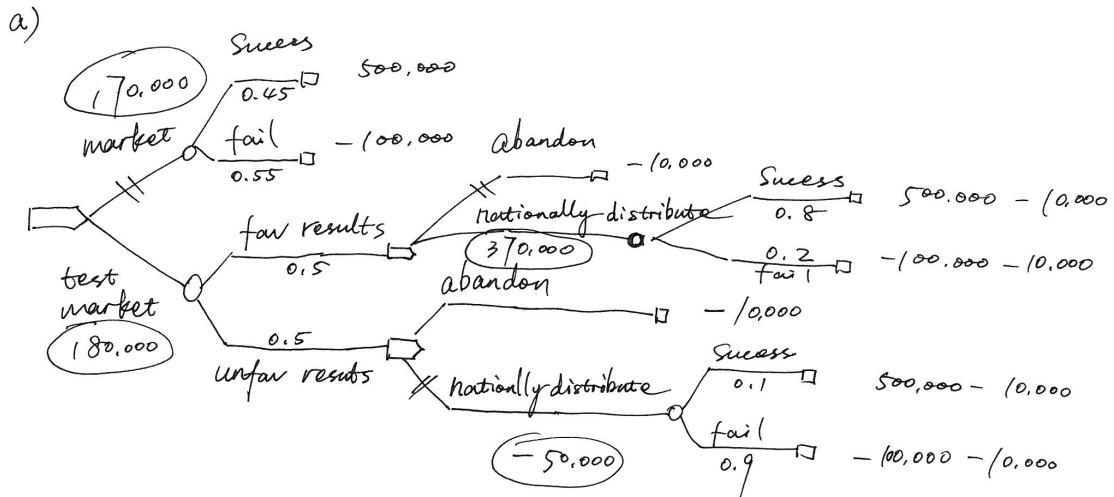
Similar calculation as for alternative A. $Y = 14.11$ and $X\% = 25.40\%$

6a.



6b. The optimal decision is B.

7. a) b) see following tree.



c) Do some test marketing, if you get a favorable result, then distribute nationally. If not, then abandon the product.

8. <http://people.brunel.ac.uk/~mastjjb/jeb/or/decmore.html>

9. $20000 * 1.03^2 * (1 + 0.04/4)^{-8} = 19594.47$.

10. $\$22,000 * (1 + d_A)^{10} = \$38,088$, hence $d_A = 5.6\%$, hence $d_R = 3.6\%$

11a. Assume discount at the end of each year. $r = 5\%$.

cost of Prius = $\$25,000 + 15000/50 * 2 * (1/r * (1 - 1/(1+r)^{10})) = \29633

cost of Miata = $\$20,000 + 15000/25 * 2 * (1/r * (1 - 1/(1+r)^{10})) = \29266

Miata is economical choice

11b. it's like assuming inflation is 3%. hence $d_r = 1.9\%$

cost of Prius = $\$25,000 + 15000/50 * 2 * (1/d_r * (1 - 1/(1+d_r)^{10})) = \30406

cost of Miata = $\$20,000 + 15000/25 * 2 * (1/d_r * (1 - 1/(1+d_r)^{10})) = \30812

Prius is economical choice

11c. now use real discount rate $r = 4\%$

$EUAC_{prius} = \$25,000 / (1/r * (1 - 1/(1+r)^{10})) + 15000/30 * 2 = \3682

$EUAC_{miata} = \$20,000 / (1/r * (1 - 1/(1+r)^{15})) + 15000/25 * 2 = \2999

Miata is economical choice.

12. Choose the metal roofing, which has the lowest EUAC. EUAC is an appropriate criterion because the buildings will be around and needing roofing for a very long time, and one can repeat each option many times, e.g. replace cheap shingles with more cheap shingles every 15 years. The EUAC for each alternative is computed as follows: let Y = lifespan in years, A = annual maintenance, and I = (purchase price + cost to install).

Then $EUAC = A + I * d / (1 - (1 + d)^{-Y})$ where d is the annual real discount rate. (We must use a real discount rate so as to have constant projected maintenance costs: the real maintenance costs stay constant while the nominal maintenance costs would increase at 3% per year.) This real discount rate $d = (1.09)/(1.03) - 1 \approx 2.9\%$. Then the EUACs are \$13.74 for cheap shingles, \$10.96 for expensive shingles, and \$9.41 for metal roofing.

13. Actual discount rate; because the nominal cashflows are constant.

14. No, they are constant.

15. yes

16. When $f < 0$ (i.e., deflation), since then $1 + f < 1$ and thus $1 + d_A = (1 + d_R)(1 + f) < 1 + d_R$.

17a. 3.7%

17b. The ones with 4yr lifespan. The EUACs are \$35.65, \$27.30, \$22.87, \$18.37, \$18.86.

18. $(1 + d) * (1 + 2\%) = 1 + 1.4\%$, $d = -0.588\%$

19. $0.99 = 1.02 * (1 + d_R)$. Hence $d_R = -2.9\%$.

20. $1m + 1m/1.04 = 1.96m$.

21. Loan payment is $A\$ C = PMT(i, 5, -25000)$. Yearly gas cost is $100 * 4 = 400$.

So, year 1: $A\$ -(C + 400 * (1 + f))$ or $R\$ -(C * (1 + f)^{-1} + 400)$

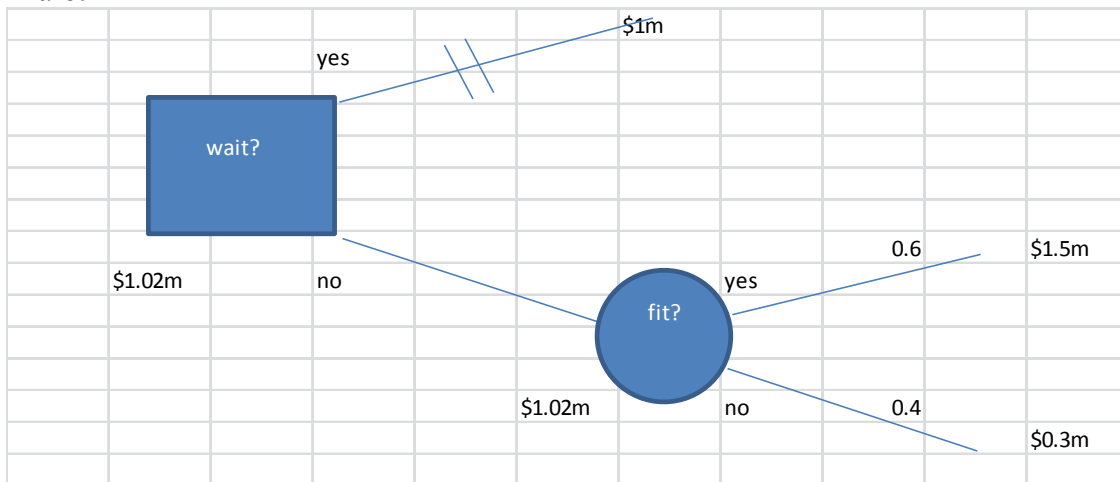
Year 2: $A\$ -(C + 400 * (1 + f)^2)$ or $R\$ -(C * (1 + f)^{-2} + 400)$

Year 3: $A\$ -(C + 900 * (1 + f)^3)$ or $R\$ -(C * (1 + f)^{-3} + 900)$

Year 4: $A\$ -(C + 400 * (1 + f)^4)$ or $R\$ -(C * (1 + f)^{-4} + 400)$

Year 5: $A\$ 12,600 * (1 + f)^5 - C$ or $R\$ 12,600 - C * (1 + f)^{-5}$.

22a-b.



22c. The optimal strategy is to not wait, and the expected profit is \$1.02m.

22d. $1.4p + 0.2(1-p) \geq 1.02$

→ $p \geq 0.683$

The information given should result in at least 68.3% chance of the case fitting the iPhone.

